Kelvin Transformations for Simulations on Infinite Domains

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01 Introduction

Whole Pipeline

INTRODUCTION

Why do we use kelvin transformation?

- Solving PDEs on **infinite domain** is a **challenging task** .
- We can treat infinite domain as **finite.**

Previous Solutions for infinite domain simulation

- Solvers relying on **discretizing the space** require the domain to be truncated into a finite size, introducing **artificial boundaries**.
- Monte-Carlo-based PDE solvers for the direct evaluation of the solution usually **involve a random walk** in the domain, which converges much more slowly as the walker likely wanders to infinity.

Whole Pipeline

02 Backgrounds

PDE, Boundary Condition, Kelvin Transformation

PDEs

1. Laplace Equation

   Pressure, Electromagnetic case

2. Poisson Equation

   General form of Laplace equation

3. Helmholtz Equation

   Wave Equation, Heat Equation

Boundary Conditions

1. Dirichlet Boundary Condition

2. Neumann Boundary Condition

3. Robin Boundary Condition

4. Sommerfeld radiating Condition

Kelvin Transformation Algorithm (1)

Algorithm 1 Poisson equation solver on an exterior domain

Input: Poisson problem (2), (3) on an infinite domain, given a boundary condition.

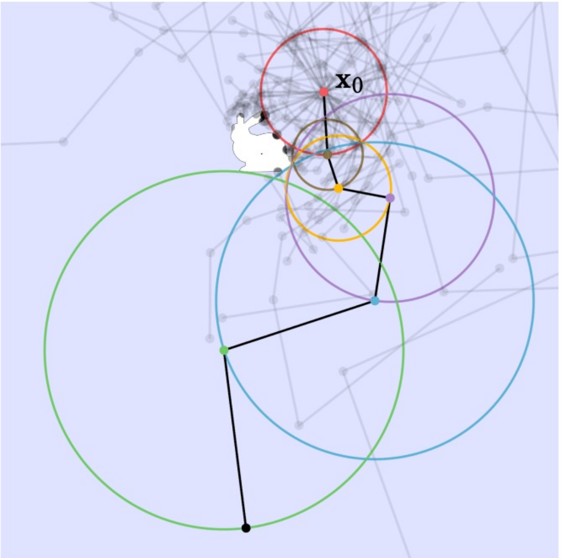
1: $V \leftarrow$ Solve the interior problem (8) with appropriate boundary conditions for V (Section 4.1.1).

2: $U(\mathbf{y}) = |\mathbf{y}|V(\mathbf{y})$.

3: $u(\mathbf{x}) = U(\frac{\mathbf{x}}{|\mathbf{x}|^2})$.

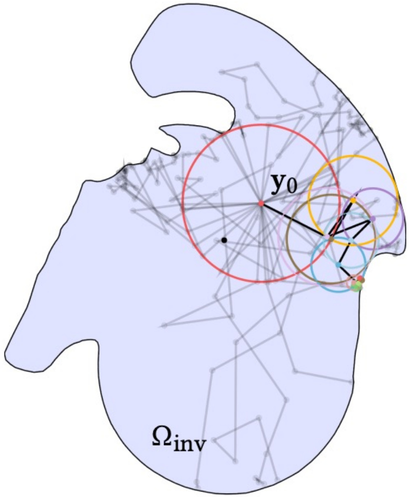
Output: $u(\mathbf{x})$.

Kelvin Transformation Algorithm (2)



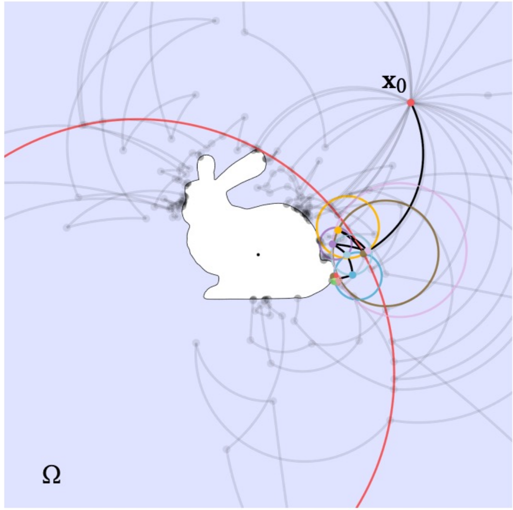
(a) WoS-RR on an infinite domain

VS



(c) WoS-KT on the inverted domain

→

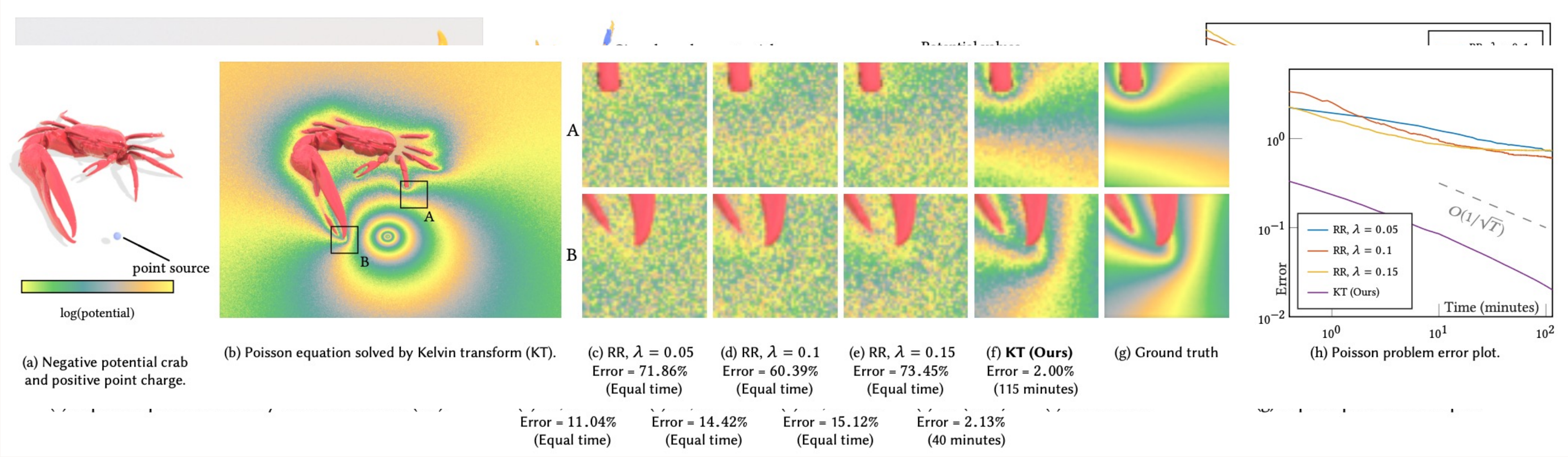


(b) WoS-KT on the original domain

03 Applications & Result

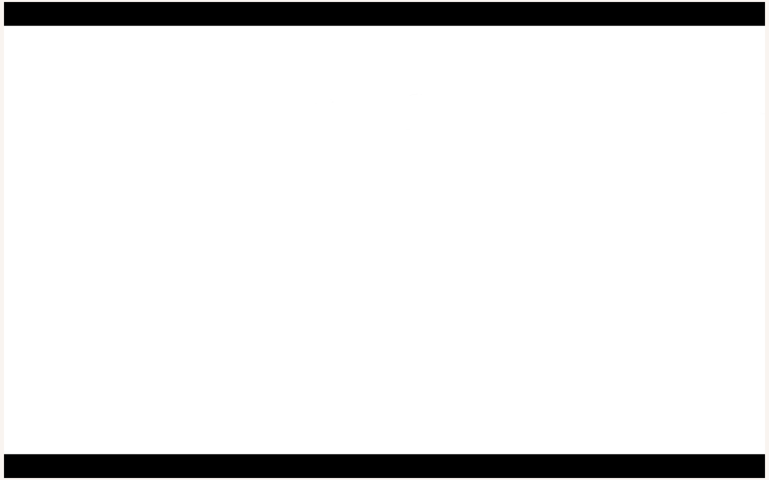
Solving Poisson, Laplace Equation, And Applying Monte Carlo

Applications (WoS-KT)

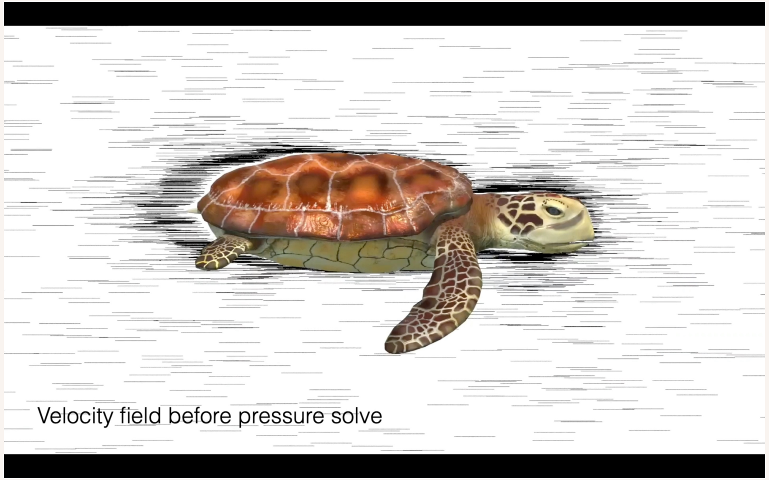


Applications (Simulation)

Magnetic flux



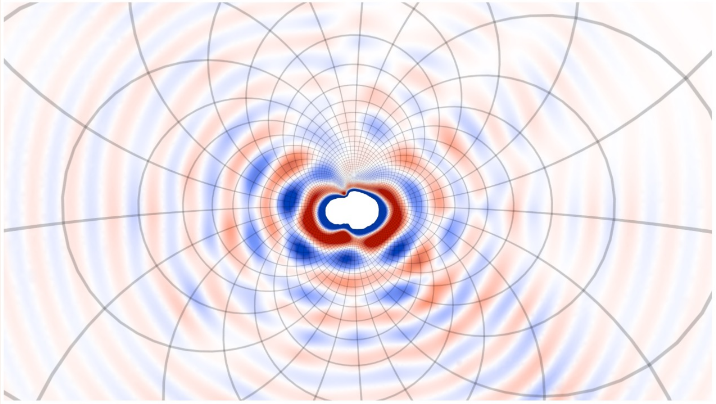
Pressure Projection



04 Helmholtz Equation

Solving it on Exterior Domains

Helmholtz Equation



- **Sommerfeld radiating condition** establish **uniqueness** of Helmholtz equation.
- **Contain {0}** as part of domain of **target PDE**.

HELMHOLTZ EQUATION

Algorithm 4 Helmholtz equation solver on an exterior domain

Input: Helmholtz problem (29), (31) on an infinite domain, given boundary condition.

1: $V \leftarrow$ Solve the interior problem (35) with appropriate boundary conditions (36).

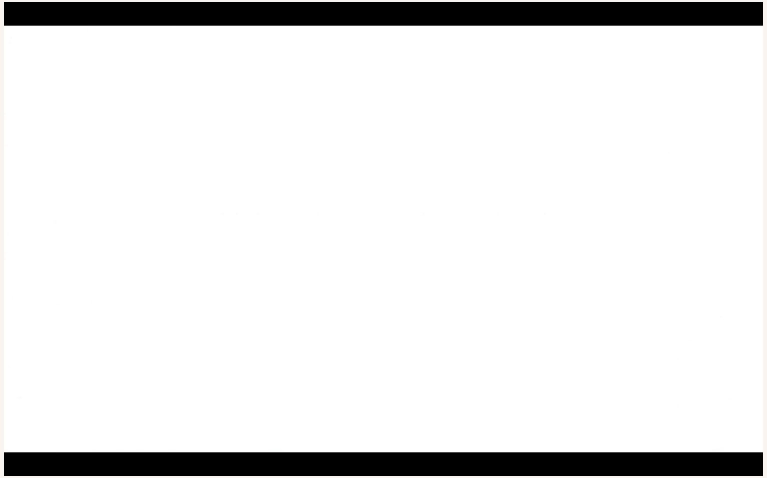
2: $U(\mathbf{y}) = G(\mathbf{x})V(\mathbf{y})$, where $G(\mathbf{x})$ is given by (32).

3: $u(\mathbf{x}) = U(\frac{\mathbf{x}}{|\mathbf{x}|^2})$.

Output: $u(\mathbf{x})$.

Helmholtz Equation

Acoustic wave



THANKS!

Do you have any questions?